

**2022 FALL REAL ANALYSIS (I) @ NCTU APPL. MATH.
HOMEWORK 4**

- Please answer the following questions in details, which means you need to state all theorems and all reasons you have been using.
 - Please mark your name, student ID, and question numbers clearly on your answer sheet. The deadline to hand in the exercise is on November 4, 2022.
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- (1) Show that simple functions, step functions, and continuous functions with compact supports are dense in $L^1(\mathbb{R}^n)$, under the graph norm $\|\cdot\|_{L^1(\mathbb{R}^n)}$.
- (2) Suppose that f is integrable, and define $f_h(x) = f(x - h)$, for $h \in \mathbb{R}^n$. Show that $\|f_h - f\|_{L^1(\mathbb{R}^n)} \rightarrow 0$ as $h \rightarrow 0$.
- (3) For any $p > 0$, $\int_E |f - f_k|^p \rightarrow 0$ as $k \rightarrow \infty$, and $\int_E |f_k|^p \leq M$ for all $k \in \mathbb{N}$. Show that $\int_E |f|^p \leq M$.
- (4) Let $y = Tx$ be a non-singular linear transformation of \mathbb{R}^n . Suppose that $\int_E f(y) dy$ exists. Show that¹

$$\int_E f(y) dy = |\det T| \int_{T^{-1}E} f(Tx) dx.$$

¹This question is a makeup question of 5(b) in the previous exercise.